The Drop Shunting Problem (DSP)

A MIXED-INTEGER LINEAR PROGRAMMING MODEL FOR THE DROP SHUNTING PROBLEM

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Introduction

Intro & Background Info

- The beverages industry is growing at a fast rate and includes alcoholic and non-alcoholic drinks
- what is growth in beverages industry
  - Increase in beverages consumption (number of customers and demand for beverages)
  - the development of the beverages supply chain
- For the beverages to reach the final consumer, the supply chain of the bottled distribution network uses freight transportation by road.
- The Beverage company considered in this study has truck and trailer combinations, but it has more trailers than trucks.
- A trailer is loaded at the depot with the bottles of beverage for several clients. A truck then takes it to the first drop off point and leaves it there for that client's stock to be offloaded and the empty bottles to be loaded.

- Subsequently, the truck moves to another trailer that has completed loading and off loading at another client and moves the trailer to its next client or takes an empty trailer back to the depot or collects another loaded trailer from the depot and takes it to a client.
- Hence, we have a pool of trucks shunting a pool of trailers around between clients and the depot.
- The drop shunting problem is one of the many challenges encountered by the beverage Industry.
- Researchers have engaged in engineering solutions to solve the vehicle routing problem.

Problem Statement

Problem Statement

- The problem therefore is to find the most efficient routing system to minimise the transportation cost in the delivery of goods.
  Question: How do we do this while travelling the least distance?
  Question What is the right number of trucks and trailers?
  Question: How should the deliveries be structured to minimise total cost?

Solution Approach

- There is also a time component associated to the DSP because each trailers have different minimum times spent at each point before the next truck can come and take it to its next destination.

Assumptions

- The minimum time would be roughly proportional to the size of the loading/unloading.
- The time frame for the trucks to move from one delivery point to another is relatively high.
- Trailers cannot stand overnight at a client and therefore they must be brought back to the depot by close of day.

Approach

- To solve the DSP, we assume that the problem is either a variant of vehicle routing problem (VRP) that includes Pickup and Delivery or that it is Job scheduling Problem (JSP).
- Proposed a Mixed-Integer Linear and Dynamic Programming Models with K-means clustering for solving the DSP.

Methodology-DSP Formulation

DSP Formulation- MVRPPD

The DSP can be formulated as a Modified-Vehicle Routing Problem(MVRP) using binary integer variables x_{ij} and d_{ij}, with the following interpretation.

variables

- x_{ij}^k : decision of taking path $i \to j$ in cluster k.
- d_{ij} : distance from node i to node j.

parameters

- S: set of locations (depot + customers)
- r: customers
- C: carrying capacity
- D: demand

Given p trucks, we assume we have p clusters.

Proposed DSP Model-MVRPPD

$$\min \sum_{k} \sum_{i=0} \sum_{j=0} d_{ij} x_{ij}^{k} \tag{1}$$

s.t

$$\sum_{k} \sum_{i} x_{ij}^{k} = 1 \qquad i \in S \setminus \{0\} \tag{2}$$

$$\sum_{k} \sum_{j} x_{ij}^{k} = 1 \qquad j \in S \setminus \{0\} \tag{3}$$

$$\sum_{k} \sum_{i} D_i x_{ij}^{k} \leq C \qquad i \in S \setminus \{0\} \tag{4}$$

DSP Formulation- JSP

The DSP can be formulated as a Job Scheduling Problem(JSP) that takes into account the max span of the job combination. The following decision variables x_{ij}^k and c_u^k and parameters were used in the formulation.

variables/parameters

- n : number of jobs (index starts from 1)
- x_{ij}^k : assignment decision variable

$$x_{ij}^k = \begin{cases} 1 & \text{if job } i \text{ is assigned right before job } j \text{ in cluster } k, \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

- p : number of trucks.
- c_u^k: span of job combination u
- $c_{\max}^k = \max \text{span}\{c_1^k, \cdots, c_u^k\}$

Proposed New DSP Mathematical Model

$$\min \quad c^k_{\max} \tag{6}$$

s.t

$$\sum_{k=0}^{p}\sum_{i=0}^{n} x^k_{ij} = 1 \tag{7}$$

$$\sum_{k=0}^{p}\sum_{j=0}^{n+1} x^k_{ij} = 1 \tag{8}$$

$$x^k_{ii} = 0 \qquad \forall i,\ \forall k \tag{9}$$

$$\sum_{k=1}^{p}\sum_{i=1}^{n}\sum_{j=1}^{n} x^k_{ij} = n \tag{10}$$

K-Mean clustering

K-means algorithm is an iterative algorithm that tries to partition the dataset into K pre-defined distinct non-overlapping subgroups (clusters) where each data point belongs to only one group.

- It tries to make the intra-cluster data points as similar as possible while also keeping the clusters as different (far) as possible

The K-mean clustering algorithm is as follows:

1. Specify number of clusters K.
2. Initialize centroids by first shuffling the dataset and then randomly selecting K data points for the centroids without replacement
3. Keep iterating until there is no change to the centroids. i.e assignment of data points to clusters isn't changing.

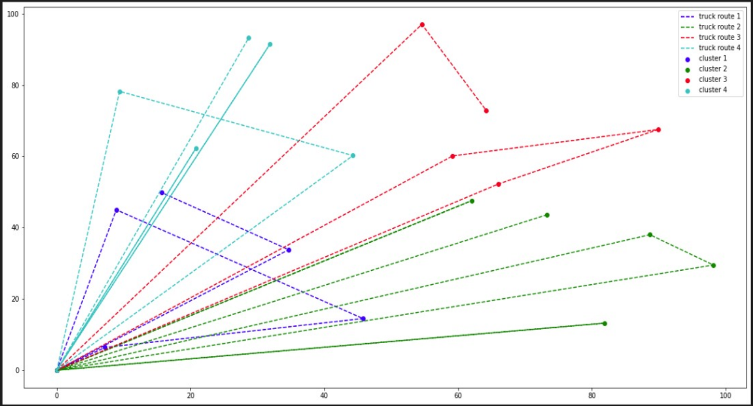
Results & Discussion

Simulated Data

	SKU_14	SKU_23	SKU_24	SKU_32	SKU_44	SKU_54	SKU_64	SKU_73	SKU_74	SKU_83	SKU_84	SKU_92	SKU_11	SKU_71	SKU_81	Routes	dist_from_depo_x	dist_from_depo_y
0	0	17000	0	0	0	0	15000	0	0	0	0	22000	0	0	0	1	46.1	16.1
1	0	0	14000	0	0	25000	12000	0	0	0	0	0	16	0	0	2	82.1	26.1
2	0	0	0	0	0	0	0	0	19000	12000	23000	0	22	0	88	3	83.8	71.8
3	0	0	0	0	0	0	14000	0	0	0	14000	0	75	0	0	1	0.1	43.8
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	55	1	8.9	38.9
5	0	0	0	0	0	0	0	0	0	0	0	10000	0	0	0	3	81.4	88.8
6	0	0	0	0	0	0	25000	0	0	22000	18000	0	0	0	0	4	20.0	70.4
7	0	0	0	0	15000	0	0	10000	0	0	0	18000	70	0	0	3	67.0	67.8
8	0	0	0	0	0	0	0	12000	0	0	0	0	0	0	0	3	66.6	78.1
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	36.1	36.6
10	0	0	12000	0	17000	0	0	0	0	0	24000	0	74	0	0	1	0.3	20.9
11	0	11000	0	0	20000	0	0	0	0	0	0	0	0	0	0	4	31.4	95.3
12	0	0	0	0	0	0	25000	0	0	0	0	0	0	41	0	1	48.0	30.6
13	0	0	0	0	0	0	17000	0	0	0	0	0	0	0	13	1	11.3	7.9
14	0	0	0	0	0	0	23000	0	0	0	0	0	0	0	0	3	86.4	91.2
15	0	21000	0	0	19000	0	20000	0	0	0	0	0	0	0	56	3	81.8	67.2
16	0	0	0	0	0	0	0	0	0	0	0	12000	0	0	47	1	35.2	42.7
17	0	0	0	0	0	17000	22000	0	11000	0	18000	0	0	0	0	3	99.1	55.5
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	62.9	89.6
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	18	1	2.3	21.5
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	81.3	95.6
21	24000	0	0	0	0	10000	0	0	20000	0	0	0	19	0	0	1	41.2	34.5
22	15000	18000	0	0	0	0	0	12000	18000	0	25000	20000	0	0	0	3	67.1	73.3
23	0	0	0	0	0	18000	0	0	23000	14000	0	0	92	0	0	1	38.6	42.9
24	0	0	12000	0	0	0	0	0	20000	0	0	0	0	0	0	4	31.4	99.2

Results

```
Route Taken by truck  1 :  [0, 1, 2, 3, 0, 4, 5]
Route Taken by truck  2 :  [0, 8, 0, 6, 7, 0, 9, 0, 10]
Route Taken by truck  3 :  [0, 14, 11, 12, 0, 13, 15]
Route Taken by truck  4 :  [0, 17, 0, 16, 0, 18, 19, 0, 20]
```



Discussion

- A shortcoming of this work was the absence of the availability of real datasets. Real datasets would surely be larger than the artificial dataset used here, and would provide better guidance as to what further work is required for this model.
- The developed dynamic programming method integrated with the K-means clustering proved useful for solving the DSP.
- One issue which could be addressed in future research is to explore other heuristic approaches like Particle Swarm Optimization for solving the problem.
- The simple MVRPPD used in this study did not capture every detail of the problem. Hence, the introduction new model JSP formulation. A future improvement would be to consider the TYPE II version of the MVRPPD.
- Although the methods introduced here are shown to work, results from an exact method need to be obtained for comparison and validation.

Conclusion

Conclusion

- Novel Mathematical models for solving the DSP have been presented.
- Tests on a small instance were used to validate the models.
- The results have shown that the our proposed model and approach works and are practical for use in industry.

Any Questions?